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## Auxetics and other systems of “negative” characteristics

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This is the jubilee, tenth issue of *physica status solidi (b)* focussed on auxetics, i.e. systems exhibiting negative Poisson's ratio (PR), and other materials of “negative” characteristics [1-9]. This time it contains 32 papers on auxetics, negative thermal expansion, negative stiffness, negative effective mass-density tensor, and other topical systems and phenomena.

The issue starts from the paper by Graeme Walter Milton on models of, so called, dilational materials [10]. The latter show PR equal to  $-1$  for finite deformations. Thus, they constitute a subclass of perfect auxetics, for which  $PR = -1$  is required at infinitesimal deformations. A few new three-dimensional (3D) models of dilational materials are presented as well as examples of perfectly auxetic systems, which are considered by the author as non-dilational. One of these non-dilational systems, known as a hexachiral auxetic, is the subject of three following papers [11-13].

In the first of them Chan Soo Ha, Eric Hestekin, Jianheng Li, Michael E. Plesha, and Roderic S. Lakes use hexachiral auxetic lattices of PR approaching  $-1$  to obtain systems of arbitrarily large, controlled positive or negative thermal expansion [11]. Examples of such systems, made from bimetallic strips by the authors, which correspond to Cosserat thermoelastic solids, indeed show highly negative thermal expansions.

Hexachiral auxetics with PR close to  $-1$  are also studied in the following two papers [12, 13]. In the first of them, by Alessandro Airoidi, Paolo Bettini, Paolo Panichelli, Mehmet F. Oktem, and Giuseppe Sala [12], the role of composite chiral ribs with auxetic behaviour is outlined and a chiral composite rib is then produced and tested. The second paper, by Alessandro Airoidi, Paolo Bettini, Paolo Panichelli, and Giuseppe Sala [13], concerns technological processes developed to produce chiral honeycombs made of thin composite laminates. A new approach is proposed. Hexachirals are manufactured and subsequently tested to assess the auxetic response. The obtained results indicate that the novel technological route provides a significant contribution for the application of composite chiral honeycombs to morphing structures.

Induced out-of-plane auxetic behavior in needle-punched commercial nonwoven fabrics is studied by Prateek Verma, Meisha L. Shofner, Angela Lin, Karla B. Wagner, and Anselm C. Griffin in their very interesting experimental paper [14]. They describe a heat-compression protocol used in the treated fabrics. Using micro-CT imaging the authors find that vertical fiber bundles/columns, produced

in the needle punching step of fabric manufacture, are tilted and buckled as a result of the heat-compression treatment. This research suggests that the reorientation of these columns during subsequent uniaxial strain drives the auxetic response.

The subject of the next two papers [15, 16] is  $\alpha$ -cristobalite, a metastable polymorph of silica which has the remarkable property of exhibiting a negative Poisson's ratio. Frank Nazaré and Andrew Alderson present and compare three analytical models of corner-sharing tetrahedra and show that the model in which two tetrahedral rotation mechanisms act concurrently with tetrahedral dilation gives predictions in very good agreement with all experimental on-axis Poisson's ratios [15].

Ruben Gatt, Luke Mizzi, Keith M. Azzopardi, and Joseph N. Grima investigate the mechanical properties and deformation mechanisms of  $\alpha$ -cristobalite through a force-field based approach [16]. They have found that the mechanical properties of the system can be accurately described through semi-rigid SiO<sub>2</sub> tetrahedra.

Stishovite, a mineral which is estimated to be abundant in the Earth's mantle and exhibits negative Poisson's ratios for a range of loading directions at specific ambient pressure ranges, is the subject of the paper by Keith M. Azzopardi, Jean-Pierre Brincat, Joseph N. Grima, and Ruben Gatt [17]. The deformations which lead to auxetic behaviour in the (001) plane are modelled by the type  $\alpha$  dilating rhombi mechanism. That work also corroborates a three-dimensional distorting octahedra mechanism [17].

Simulations using particle methods are the subject of the following four papers. The auxeticity of face-centered cubic metal (001) nanoplates is studied by Duc Tam Ho, Hokun Kim, Soon-Yong Kwon, and Sung Youb Kim who present results of an atomistic study [18]. They find that the behaviour of the PRs of metal nanoplates is strongly dependent on the characteristics of a phase transformation that takes place in their bulk counterparts as well as on the amount of compressive stress induced in the nanoplates. The simulations indicate also that surface stresses as well as the critical strains for the phase transformation strongly influence the PR of (001) nanoplates. Moreover, strongly auxetic nanoplates can be obtained by raising the temperature.

Novel nanostructures based on graphene sheets are investigated via molecular dynamics simulations by Julia A. Baimova, Leysan Kh. Rysaeva, Bo Liu, Sergey V. Dmitriev, and Kun Zhou [19]. They show that the structural and mechanical properties of bulk carbon nanomaterials can be altered by severe plastic shear deformation. They also show that shear strain leads to the formation of stable structures, even at relatively small strain.

Jakub W. Narojczyk, Paweł M. Pięłowski, Krzysztof W. Wojciechowski, and Konstantin V. Tretiakov report Monte Carlo simulations of mono- and polydisperse two-dimensional crystals interacting through hard-core repulsive Yukawa potential. In contrast to its three-dimensional version which is partially auxetic, the studied system is non-auxetic, i.e., shows positive Poisson's ratio.

Scaling of Lennard-Jones (LJ) liquid elastic moduli, viscoelasticity and other properties along fluid-solid coexistence are determined using Molecular Dynamics simulation by David M. Heyes, Daniele Dini, and Arkadiusz C. Branka [21]. They show that various properties of the system are essentially invariant or structurally isomorphic along this line when scaled by so-called macroscopic variables [21]. A generalized Cauchy relationship in which the infinite frequency longitudinal modulus is proportional to the longitudinal modulus of the fluid was found to be obeyed very well for the LJ fluid phase along this coexistence line.

In the next six papers the finite element method (FEM) is applied to study auxetic properties of models of simple systems and models of composites. The effects of Poisson's ratio on the deformation of thin membrane structures under indentation are studied by Jensen Aw, Hongyi Zhao, Andrew Norbury, Lisa Li, Glynn Rothwell, and James Ren [22]. They show that negative Poisson's ratios have direct influence on the membrane deformation domain, including the force-displacement curve, the deflection profile and the contact area. Critical factors affecting the deformation mechanisms are discussed with reference to potential use of the Poisson's ratio effects.

Cellular plates with auxetic rectangular perforations are considered by Alexander Slann, William White, Fabrizio Scarpa, Katarzyna Boba, and Ian Farrow [23]. They show that the rectangular voids used in this structure produce a rigid rotating squares effect. Through numerical modeling and experimental testing they confirm auxetic behaviour of the structure, with increased negative Poisson's ratio values and compare it to rhomboidal patterns of perforations available in the open literature. The rhomboidal patterns provide more negative Poisson's ratio at small porosity level, for the rest – the two structures provide equivalent auxeticity.

Tomasz Strek, Hubert Jopek, and Maria Nienartowicz simulate the effective properties and dynamic response of a sandwich panel made of two face-sheets and an auxetic core [24]. They show that it is possible to create auxetic sandwich panels made of two solid materials with positive Poisson's ratio. This is even possible if the filler material is nearly incompressible, but can move in an out-of-plane direction. Moreover, the effective Young's modulus of such sandwich panels becomes very large if the Poisson's ratio of the filler material tends to  $-1$ .

A unidirectional fibrous composite built of two constituent materials of different thermo-mechanical properties is studied by Hubert Jopek and Tomasz Strek [25]. They show that, by a proper choice of both the geometry of fibres and their thermo-mechanical properties, composites of required properties can be obtained.

Luke Mizzi, Ruben Gatt, and Joseph N. Grima consider auxetic systems created through the introduction of elliptical grooves, meant to mimic the rotating units mechanism [26]. They find that those systems have a potential to exhibit Poisson's ratios ranging from ca.  $-1$  to the Poisson's ratio of the material of the

system itself, with some systems also possessing the remarkable property of a zero Poisson's ratio.

Xiaonan Hou and Hong Hu introduce a composite structure with tunable mechanical properties [27]. The composite consists of a matrix material and a stack of inclusions. Due to a special deformation mechanism of the inclusions, the global structure exhibits unusual properties over a wide range of applied deformations. The specific properties of that composite structure can be tailored by controlling the geometrical features and density of the inclusions.

The next two papers exploit analytical methods to study deformations of auxetics. Teik-Cheng Lim analyses the elastic stability of auxetic columns using third order shear deformation theory [28]. The advantage of that approach is that, in contrast to other methods, it does not require any shear correction factor. The detailed analysis of elastic stability of isotropic columns with special emphasis on auxetic ones is presented.

Investigations of the elastic properties of 6-constant tetragonal nano/microtubes are presented by Robert V. Goldstein, Valentin A. Gorodtsov, Dmitry S. Lisovenko, and Mikhail A. Volkov [29]. Analytical formulae for Young's modulus and Poisson's ratio are obtained in the case of axial deformations. Parametric dependences of Young's modulus and Poisson's ratios for nano/microtubes are analyzed in detail.

Wave propagation in auxetics is the subject of the following four papers. T. Bui Dinh, Van Cao Long, and Krzysztof W. Wojciechowski present exact analytical solutions and numerical simulations of solitary waves in auxetic rods with quadratic nonlinearity [30]. Using the, so called, F-expansion method, a new class of traveling solitary waves is obtained. Some numerical simulations are compared with the obtained exact analytical solutions. The comparison indicates that Secant pulses generate solitary waves closer to the analytical predictions than Gaussian pulses. This is similar to the case of the variational method, where the Secant trial functions also appear to be more proper than the Gaussian ones.

Propagation of surface waves and surface resonances along cylindrical cavities in materials with any allowed Poisson's ratio is discussed in two papers by Paweł Sobieszczyk, Mirosław Gałązka, Dominik Trzupek, and Piotr Zieliński [31, 32]. For a clean surface [31] propagation modes with complex frequencies, i.e. with finite life times are found. It is shown that a strong radial component of the longitudinal resonance occurs at wavelengths comparable to the cavity's radius especially in the incompressible limit. For thin-walled coating [32] analytical expressions are obtained for the cut-offs and for the asymptotic behaviours of the excitations in the long-wavelength and short-wavelength limits. The local surface densities of states are also calculated.

The same authors and Teik-Cheng Lim derive the Helmholtz wave equation for longitudinal waves in an elastic plate of arbitrary thickness placed in a rigid gantry ensuring a constant width [33]. The whole range of Poisson's ratio allowed for isotropic elastic media constrained in this way is considered. The

results of the presented analysis provide guidelines for designing devices aimed at a passive control of propagation of longitudinal waves in thin-walled structures.

Sławomir Czarnecki and Paweł Wawruch analyse an optimal distribution of material characterized by the Young's modulus and Poisson's ratio which maximizes the overall stiffness of an inhomogeneous and locally isotropic elastic 3D body transmitting a given surface loading to a given support [34]. It is proven that isotropic composite materials forming the bodies of extremely high stiffness exhibit negative Poisson ratio in large subdomains, which points at the significance of the auxetic material in modern structural design.

Tom Allen, Jonathan Shepherd, Trishan M. Hewage, Terry Senior, Leon Foster, and Andrew Alderson describe the low-kinetic energy impact response of auxetic and conventional open-cell polyurethane foams [35]. The auxetic samples displayed a 6 times reduction in peak acceleration, showing potential in impact protector devices such as shin or thigh protectors in sports equipment applications.

The next two papers concern negative stiffness or compressibility. Yun-Che Wang, Chih-Chin Ko, and Keng-Wei Chang consider composite materials containing negative-stiffness inclusions embedded in positive-stiffness matrix [36]. Using the finite element method under the quasi-static assumption they calculate effective coupled-field properties and determine stability boundaries. They show that, in the viscoelastic case, inclusion shapes have no effects on stability. They also conclude that insulated inclusions may cause charge accumulation at the inclusion-matrix interface and boundary surface effects may serve as stabilizing agents to the composite system.

Carbon allotropes exhibiting negative linear compressibility, i.e. the phenomenon of expansion rather than shrinkage in at least one direction upon the application of a hydrostatic compressive pressure, are studied by Jean Paul Formosa, Reuben Cauchi, and Joseph N. Grima [37]. Through analysis of some literature data and through static force-field based simulations they show that it is possible to achieve this property in the novel carbon allotropes built from  $sp^2$  and  $sp^3$  hybridised carbon atoms which have a 2D projection that resembles a honeycomb motif in their (001) plane. This is in accordance with earlier predictions that honeycombs deforming through a hinging-like mechanism could exhibit this property for certain geometries.

Nonlinear positive or negative thermal expansion and equation of state of a chain with longitudinal and transverse vibrations is studied by Vitaly A. Kuzkin and Anton M. Krivtsov [38]. Using series expansions of pressure and thermal energy with respect to deformations of the bonds caused by thermal motion, they derive analytically equations of state. They also formulate a necessary and sufficient condition for negative thermal expansion at low thermal energies. They show that in the vicinity of the deformation, corresponding to zero Grueneisen parameter, the chain demonstrates negative thermal expansion at low temperatures and positive thermal expansion at higher temperatures. The

theoretical analysis is supported by the results of molecular dynamics simulations.

Tiemo Bückmann, Muamer Kadic, Robert Schittny, and Martin Wegener discuss mechanical metamaterials with anisotropic and negative effective mass-density tensor made from one constituent material [39]. At present, such structures can be fabricated by using 3D printing. Hence, the authors use numerical phonon band structure calculations to study what performance can be expected with current fabrication technology.

Compressive and dissipative behaviour of a new material, known as metal rubber, under constraints is discussed by Yanhong Ma, Di Gao, Dayi Zhang, and Jie Hong [40]. It is known that, after appropriate processing, these unusual materials can exhibit negative Poisson's ratio. Here, the authors show that the stiffness and loss factor of metal rubber increase significantly with increasing constraints.

The paper closing this issue concerns, so called, discrete breathers (DB) in 2D and 3D crystals [41]. These are spatially localized, large-amplitude vibrational modes in defect-free nonlinear lattices. Sergey V. Dmitriev, Alexander P. Chetverikov, and Manuel G. Velarde briefly describe the existence conditions of DB in crystals and present the recent results on DB in 2D crystals such as graphene and graphane and in 3D crystals such as alkali halide crystals and pure metals. The possible role of DB in solid state physics and materials science is also discussed in that paper.

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